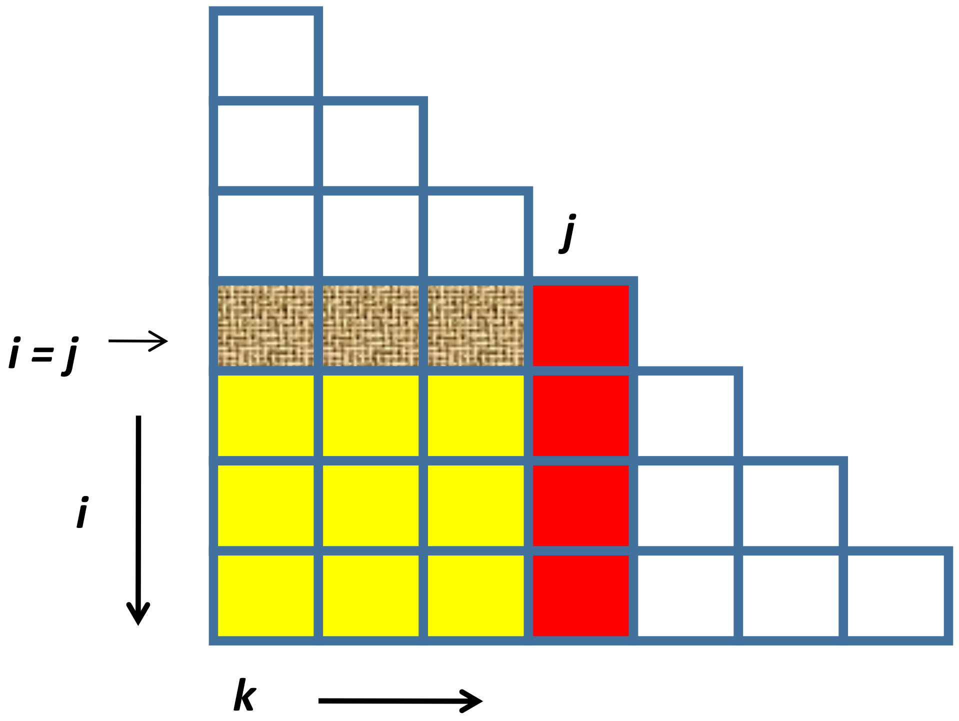
# Faktoryzacja macierzy rzadkich

# Metoda looking left.
# Macierz gęsta. Algorytm.




do j=1, N
- update column j

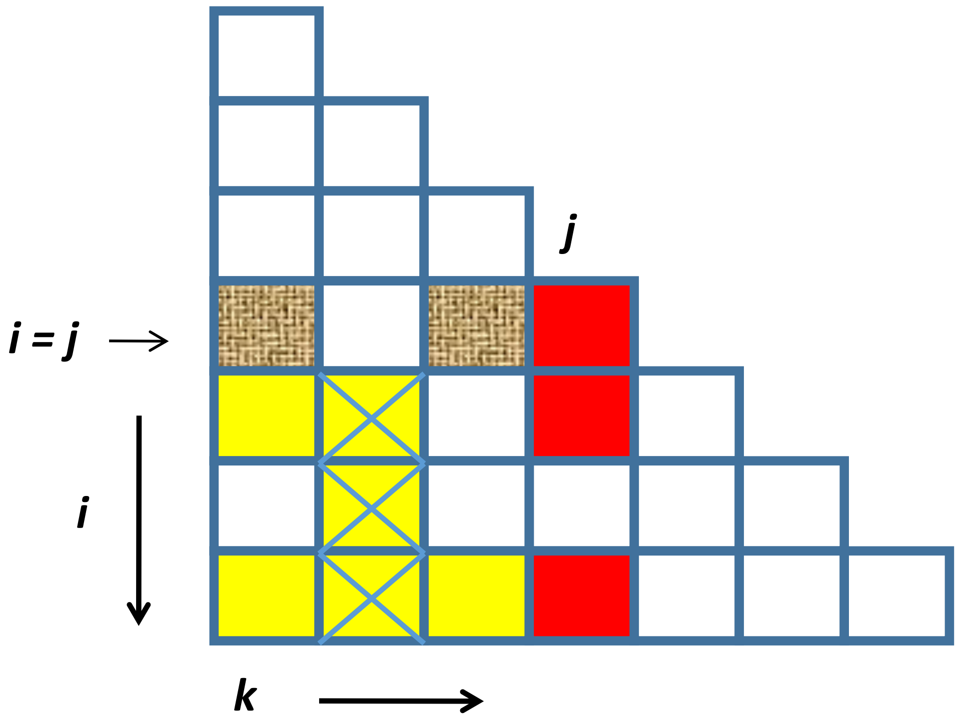
    do k=1, j-1
        do i=j, N
            $a_{ij} = a_{ij} - a_{jk} \cdot a_{ik}$
        end do
    end do

- factorize column j

    $l_{jj} = \sqrt{a_{jj}}$
    do i=j+1,N
        $l_{ij} = a_{ij}/l_{jj}$
    end do

end do

# Metoda looking left.
# Macierz rzadka. Algorytm.




```
do j=1, N
    ▪ update column j
    do k ϵ List[j]
        do i ϵ L_k
            a_ij = a_ij - l_jk·l_ki
        end do
    end do
    ▪ factorize column j
    l_jj = √a_jj
    do i ϵ L_j
        l_ij = a_ij/l_jj
    end do
end do
```

# Metoda looking left.
# Macierz rzadka. Algorytm.

**Poprawiamy elementy kolumny j: kolumny, umieszczone od lewej strony, poprawiają kolumnę j, jeśli mają niezerowy element o indeksie $l_{jk}$ :**

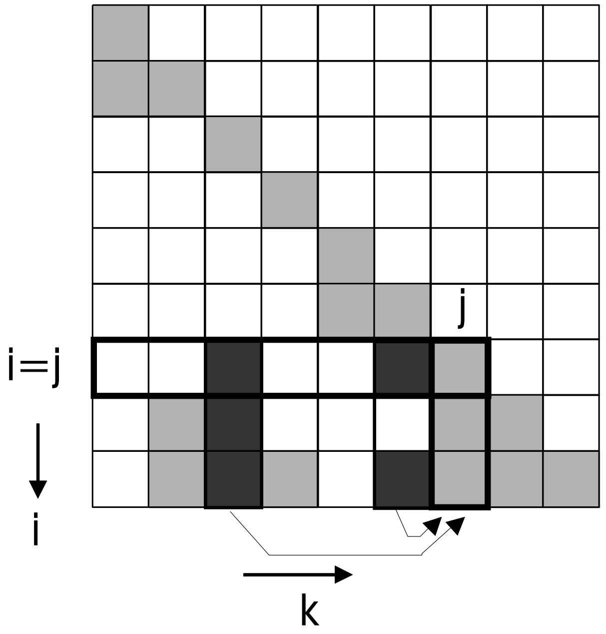
$$\mathbf{s}_j = \mathbf{s}_j - \mathbf{t}_j, \quad gdzie \quad \mathbf{t}_j = \sum_{k \in List[j]} l_{jk} \cdot \mathbf{s}_k,$$

*List*[*j*] – **zbiór indeksów k, które odpowiadają niezerowym elementom w wierszu** *i* = *j* **.**

**t – gęsty wektor.**

**W wektorze $s_k$ (*k* – numer kolumny), są utrzymane niezerowe elementy i > j (poniżej diagonali).**

**Poprawienie obejmuje tylko elementy kolumny j, które odpowiadają niezerowym elementom kolumny *k*.**



# Metoda looking left.
# Macierz rzadka. Przykład.

$$
\begin{pmatrix}
121 & & s & & \\
0 & 256 & & y & \\
11 & 16 & 38 & & m \\
0 & 0 & 0 & 81 & \\
-11 & -16 & 10 & 9 & 32
\end{pmatrix}
\Rightarrow
\left(\begin{array}{c|cccc}
11 & & & & \\
0 & 256 & & & \\
1 & 16 & 38 & & \\
0 & 0 & 0 & 81 & \\
-1 & -16 & 10 & 9 & 32
\end{array}\right)
\Rightarrow
\left(\begin{array}{cc|ccc}
11 & & & & \\
0 & 16 & & & \\
\hline
1 & 1 & 38 & & \\
\hline
0 & 0 & 0 & 81 & \\
-1 & -1 & 10 & 9 & 32
\end{array}\right)
\Rightarrow
\left(\begin{array}{ccc|cc}
11 & & & & \\
0 & 16 & & & \\
1 & 1 & 6 & & \\
0 & 0 & 0 & 81 & \\
-1 & -1 & 2 & 9 & 32
\end{array}\right)
\Rightarrow
$$

$$
\mathbf{t}_3 = (1)\cdot\begin{pmatrix}1\\0\\-1\end{pmatrix} + (1)\cdot\begin{pmatrix}1\\0\\-1\end{pmatrix} = \begin{pmatrix}2\\0\\-2\end{pmatrix}; \quad \hat{\mathbf{s}}_3 = \mathbf{s}_3 - \mathbf{t}_3 = \begin{pmatrix}38\\0\\10\end{pmatrix} - \begin{pmatrix}2\\0\\-2\end{pmatrix} = \begin{pmatrix}36\\0\\12\end{pmatrix};
$$

$$
\left(\begin{array}{cccc|c}
11 & & & & \\
0 & 16 & & & \\
1 & 1 & 6 & & \\
\hline
0 & 0 & 0 & 9 & \\
\hline
-1 & -1 & 2 & 1 & 32
\end{array}\right)
\Rightarrow
\left(\begin{array}{ccccc}
11 & & & & \\
0 & 16 & & & \\
1 & 1 & 6 & & \\
0 & 0 & 0 & 9 & \\
\hline
-1 & -1 & 2 & 1 & 5 \\
\hline
\end{array}\right)
$$

$$
\mathbf{t}_5 = (-1)^2 + (-1)^2 + 2^2 + 1^2 = 7; \quad \hat{\mathbf{s}}_5 = \mathbf{s}_5 - \mathbf{t}_5 = 32 - 7 = 25;
$$

Dla elementu diagonalnego w wierszu j:

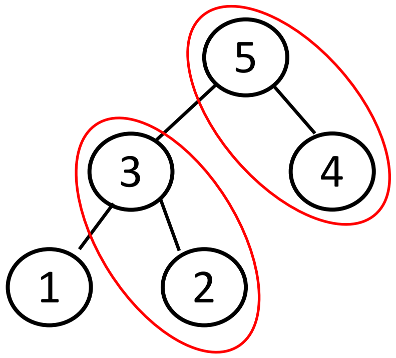
$$
l_{jj} = \sqrt{a_{jj} - \sum_{k \in List[j]} l_{jk}^2}
$$

# Metoda wielofrontalna.
# Macierz rzadka. Przykład 1.

$$
\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{pmatrix}
121 & & s & & \\
0 & 256 & & y & \\
11 & 16 & 38 & & m \\
0 & 0 & 0 & 81 & \\
-11 & -16 & 10 & 9 & 32
\end{pmatrix}
\end{array}
$$

Perm = {1, 2, 3, 4, 5}

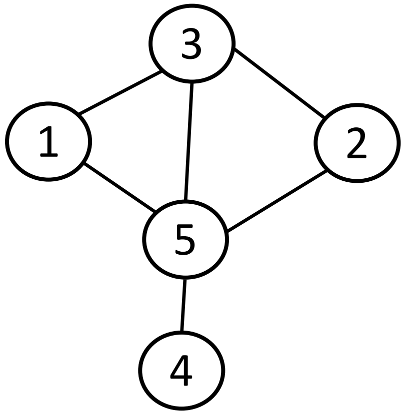
Drzewo eliminacji



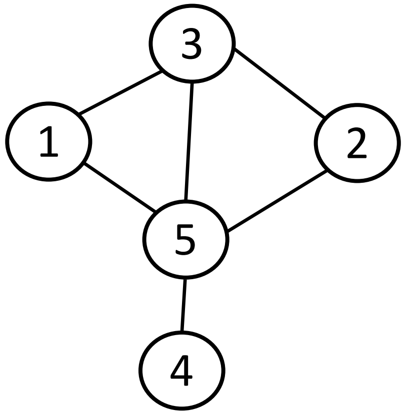

1. Analiza struktury macierzy rzadkiej

Struktura macierzy sfaktoryzowanej

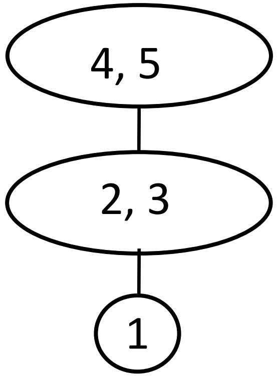
Graf przyległości




Faktor-graf



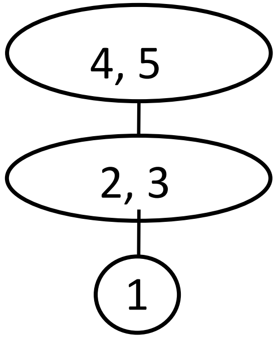

Drzewo superwęzłowe



# Metoda wielofrontalna. Macierz rzadka. Przykład 1.

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 121 & & s & & \\
2 & 0 & 256 & & y & \\
3 & 11 & 16 & 38 & & m \\
4 & 0 & 0 & 0 & 81 & \\
5 & -11 & -16 & 10 & 9 & 32
\end{array}
$$

Drzewo superwęzłowe




## 2. Faktoryzacja numeryczna

$$\begin{pmatrix} \mathbf{A} & \mathbf{W}^T \\ \mathbf{W} & \mathbf{U}_0 \end{pmatrix} = \begin{pmatrix} \mathbf{L} & 0 \\ \tilde{\mathbf{W}} & \mathbf{U} \end{pmatrix} \begin{pmatrix} \mathbf{L}^T & \tilde{\mathbf{W}}^T \\ 0 & \mathbf{I} \end{pmatrix};$$

$$\mathbf{A} = \mathbf{L} \cdot \mathbf{L}^T; \quad \mathbf{L}\tilde{\mathbf{W}}^T = \mathbf{W}^T \rightarrow \tilde{\mathbf{W}}^T; \quad \mathbf{U} = \mathbf{U}_0 - \tilde{\mathbf{W}}\tilde{\mathbf{W}}^T.$$

$\mathbf{W}^T$, $\mathbf{L}$

$$\mathbf{F}_1 = \begin{array}{c} 1 \\ 3 \\ 5 \end{array}\begin{pmatrix} 121 & 11 & -11 \\ 11 & 0 & 0 \\ -11 & 0 & 0 \end{pmatrix}; \quad \mathbf{C}_1 = \begin{array}{c} 1 \\ 3 \\ 5 \end{array}\begin{pmatrix} 11 \\ 1 \\ -1 \end{pmatrix}; \quad \mathbf{U}_1 = \begin{array}{c} 3 \\ 5 \end{array}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \end{pmatrix} = \begin{array}{c} 3 \\ 5 \end{array}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}.$$

$\mathbf{U}_0$, $\tilde{\mathbf{W}}$

$$\mathbf{F}_{4,5} = \begin{array}{c} 4 \\ 5 \end{array}\begin{pmatrix} 81 & 9 \\ 9 & 32 \end{pmatrix} + 5(-6) = \begin{array}{c} 4 \\ 5 \end{array}\begin{pmatrix} 81 & 9 \\ 9 & 26 \end{pmatrix};$$

$$\mathbf{C}_{4,5} = \begin{array}{c} 4 \\ 5 \end{array}\begin{pmatrix} 9 & 0 \\ 1 & 5 \end{pmatrix};$$

$\mathbf{A}$, $\mathbf{W}^T$, $\mathbf{L}$

$$\mathbf{F}_{2,3} = \begin{array}{c} 2 \\ 3 \\ 5 \end{array}\begin{pmatrix} 256 & 16 & -16 \\ 16 & 38 & 10 \\ -16 & 10 & 0 \end{pmatrix} + \begin{array}{c} 3 \\ 5 \end{array}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{array}{c} 2 \\ 3 \\ 5 \end{array}\begin{pmatrix} 256 & 16 & -16 \\ 16 & 37 & 11 \\ -16 & 11 & -1 \end{pmatrix}; \quad \mathbf{C}_{2,3} = \begin{array}{c} 2 \\ 3 \\ 5 \end{array}\begin{pmatrix} 16 & 0 \\ 1 & 6 \\ -1 & 2 \end{pmatrix};$$

$\mathbf{U}_0$, $\tilde{\mathbf{W}}$

$$\begin{pmatrix} 16 & 0 \\ 1 & 6 \end{pmatrix}\begin{pmatrix} w_{11} \\ w_{21} \end{pmatrix} = \begin{pmatrix} -16 \\ 11 \end{pmatrix} \rightarrow \begin{pmatrix} 16w_{11} = -16 \\ w_{11} + 6w_{21} = 11 \end{pmatrix} \rightarrow \begin{pmatrix} w_{11} = -1 \\ w_{11} + 6w_{21} = 11 \end{pmatrix} \rightarrow \begin{pmatrix} w_{11} = -1 \\ w_{21} = 2 \end{pmatrix}$$

$$\mathbf{U}_{2,3} = (-1) - \begin{pmatrix} -1 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = (-1) - (5) = 5(-6);$$

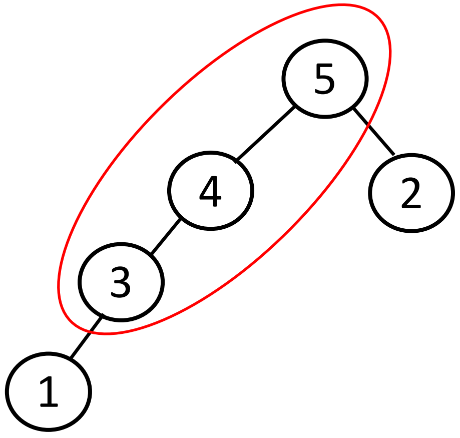
$$\mathbf{L}_{glob} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{pmatrix} 11 & & & & \\ & 16 & & & \\ 1 & 1 & 6 & & \\ & & & 9 & \\ -1 & -1 & 2 & 1 & 5 \end{pmatrix}$$

# Metoda wielofrontalna.
# Macierz rzadka. Przykład 2.

$$
\begin{array}{c}
\\
1\\ 2\\ 3\\ 4\\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array}\\
\begin{pmatrix}
25 & & s & & \\
0 & 81 & & y & \\
5 & 0 & 5 & & m \\
10 & 0 & \# & 30 & \\
0 & 9 & -2 & 6 & 19
\end{pmatrix}
\end{array}
$$

Perm = {1, 2, 3, 4, 5}

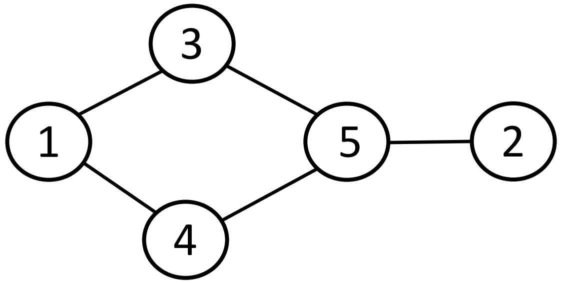
Drzewo eliminacji



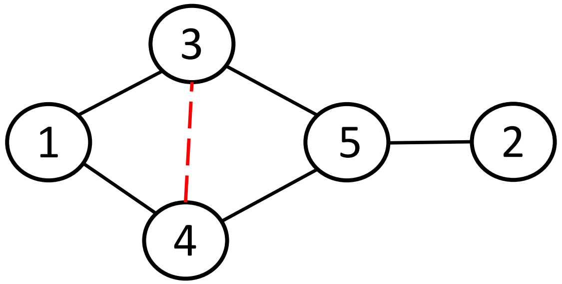

1. Analiza struktury macierzy rzadkiej

Struktura macierzy sfaktoryzowanej

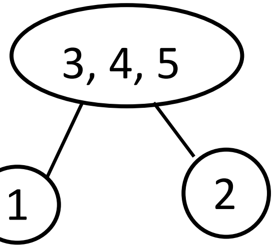
Graf przyległości




Faktor-graf




Drzewo superwęzłowe

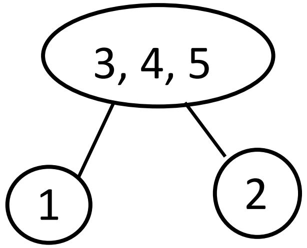


# Metoda wielofrontalna. Macierz rzadka. Przykład 2.

## 2. Faktoryzacja numeryczna

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 25 & & s & & \\
2 & 0 & 81 & & y & \\
3 & 5 & 0 & 5 & & m \\
4 & 10 & 0 & \# & 30 & \\
5 & 0 & 9 & -2 & 6 & 19
\end{array}
$$

Drzewo superwęzłowe




$$
\mathbf{F}_1 = \begin{array}{c} 1 \\ 3 \\ 4 \end{array}
\begin{pmatrix} \boxed{25} & \boxed{5 \quad 10} \\ 5 & 0 \quad 0 \\ 10 & 0 \quad 0 \end{pmatrix}
\begin{array}{l} \mathbf{A},\ \mathbf{W}^T \\ \\ \mathbf{W},\ \mathbf{U}_0 \end{array};
\quad
\mathbf{C}_1 = \begin{array}{c} 1 \\ 3 \\ 4 \end{array}\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}\ \begin{array}{l}\mathbf{L} \\ \\ \tilde{\mathbf{W}}\end{array};
$$

$$
\mathbf{U}_1 = \begin{array}{c} 3 \\ 4 \end{array}\underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}_{\mathbf{U}_0} - \begin{array}{c} 3 \\ 4 \end{array}\underbrace{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}_{\tilde{\mathbf{W}}}\underbrace{\begin{pmatrix} 1 & 2 \end{pmatrix}}_{\tilde{\mathbf{W}}^T} = \begin{array}{c} 3 \\ 4 \end{array}\begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix};
$$

$$
\mathbf{F}_2 = \begin{array}{c} 2 \\ 5 \end{array}\begin{pmatrix} 81 & 9 \\ 9 & 0 \end{pmatrix};
\quad
\mathbf{C}_2 = \begin{array}{c} 2 \\ 5 \end{array}\begin{pmatrix} 9 \\ 1 \end{pmatrix};
\quad
\mathbf{U}_2 = 5(0) - 5(1)\cdot(1) = 5(-1);
$$

$$
\mathbf{L}_{glob} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{pmatrix} 5 & & & & \\ & 9 & & & \\ 1 & & 2 & & \\ 2 & & -1 & 5 & \\ & 1 & -1 & 1 & 4 \end{pmatrix}
$$

$$
\mathbf{F}_{3,4,5} = \begin{array}{c} 3 \\ 4 \\ 5 \end{array}\overbrace{\begin{pmatrix} 5 & 0 & -2 \\ 0 & 30 & 6 \\ -2 & 6 & 19 \end{pmatrix}}^{\mathbf{A}} + \begin{array}{c} 3 \\ 4 \end{array}\overbrace{\begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix}}^{\mathbf{U}_1} + \overbrace{5(-1)}^{\mathbf{U}_2} = \begin{array}{c} 3 \\ 4 \\ 5 \end{array}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 26 & 6 \\ -2 & 6 & 18 \end{pmatrix};
\quad
\mathbf{C}_{3,4,5} = \begin{array}{c} 3 \\ 4 \\ 5 \end{array}\begin{pmatrix} 2 & 0 & 0 \\ -1 & 5 & 0 \\ -1 & 1 & 4 \end{pmatrix};
$$